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Stationary rescaled pulse in alternately concatenated fibers with $O(1)$-accumulated nonlinear perturbations

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Pulse propagation in a transmission line comprising alternately concatenated fibers with $O(1)$-accumulated perturbations of self-phase modulation and anomalous dispersion is studied. In such a line, a pulse compression process is inevitable. In certain conditions, we have found that a compressed pulse can be rescaled to the initial pulse, and hence that there exists a stationary rescaled pulse (SRP), which is distinct from other nonlinear stationary pulses such as the guiding-center, dispersion-managed, or split-step solitons. The properties of SRP are studied. We apply the rescaling to the transmission line rather than to the pulse, and we experimentally observe a SRP in a periodically rescaled transmission line.

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The nonlinear Schrödinger (NLS) equation and its stationary solutions, such as solitons, have been extensively studied in various fields of nonlinear wave guides, plasmas, fluids dynamics, and condensed matter physics. Fiber optics is one of the most important applications, and studies of nonlinear stationary pulses propagating through optical fibers with periodic structure have attracted great interest because such pulses are useful for high-speed optical soliton transmission [1–4], or mode-locked lasers [5,6]. They are discussed as stationary solutions of the NLS equation with periodically varying coefficients representing dispersion and nonlinearity of transmission lines. It is interesting to distinguish such solutions in a sense of perturbations that are induced by the transmission line and affect pulse propagation. The guiding-center (GC) soliton [1] or the dispersion-managed (DM) soliton [2] have been discovered, and they have been analyzed with a focus on the magnitude of the perturbations [7,8]. According to the GC theory [7], for the existence of the GC soliton, the magnitude of oscillating dispersion and/or nonlinearity is allowed to be $O(1)$, but their accumulated values within a period of the oscillation should be small as $O(\varepsilon)$ in the normalized space, where ε is a positive small constant. Meanwhile, in the DM theory [8], only dispersion is allowed to accumulate up to $O(1)$, and accumulated nonlinearity should be small as $O(\varepsilon)$.

In general, alternate accumulations of self-phase modulation (SPM) and anomalous dispersion cause deformation of both the time and the spectral wave form of the pulse. This principle can be used for optical pulse compression, and the comblike dispersion profiled fiber (CDPF) has been proposed [9]. In nonlinear and dispersive fibers in the CDPF, the SPM and the dispersion effects are allowed to accumulate up to $O(\varepsilon)$, respectively, and the pulse, being compressed during its propagation, is characterized by the GC theory. On the other hand, the split-step system, in which both of anomalous dispersion and SPM alternately accumulated up to $O(1)$ in a period of the transmission line, has been recently proposed and a periodic stationary pulse therein, the so-called split-step soliton, has also been found [3,10]. The wave form of the split-step soliton varies within a period of the line but periodically recovers its stationary shape. According to Ref. [10], behavior of the split-step soliton can be analyzed by the variational method with a sech ansatz having linear chirp. However, such an analysis is applicable only when the magnitude of the accumulated SPM is not greater than $O(1)$, because an intensive SPM effect induces nonlinear chirp of the pulse and the sech ansatz would be no longer valid. Therefore, numerical study rather than variational analysis should be applied.

Let us then consider pulse propagation in a transmission line where SPM and dispersion effects are alternately accumulated to an extensive magnitude of $O(1)$. Pulse compression process is inevitable in such a line. Let us think of the following rescaling of pulse $u(t)$,

$$u_{\mathrm{res}}(t) = R^{-1/2}u(t/R), \tag{1}$$

where R is a real rescaling constant. If an optical pulse $u_0(t)$ were compressed to $u(t)$ by a factor of R in a transmission line and the rescaled pulse $u_{\mathrm{res}}(t)$ calculated from Eq. (1) coincided with $u_0(t)$, and if it could propagate over arbitrary number of periods of the transmission line in a stationary manner, $u_0(t)$ would then be regarded as a stationary rescaled pulse of the transmission line. We report in this paper that it is in fact possible to identify such a stationary rescaled pulse (SRP) in alternately concatenated optical fibers where the perturbations such as SPM and anomalous dispersion effects alternately accumulate up to $O(1)$. There are many contributions on nonlinear pulse propagation in periodic transmission lines with multisegmented fibers [1–11], and it is the fact that our proposed transmission line here is a generalization of the split-step system in the sense of extensive magnitude of the accumulation of nonlinearity. Indeed, SRP shows qualitatively distinct characteristics from the fundamental soliton, GC soliton, DM soliton, and split-step soliton, because of introduction of the rescaling as well as different magnitude of the accumulated perturbations. As we shall discuss later, the rescaling of the pulse is mathematically equivalent to the rescaling of the transmission line. The rescaling of the trans-

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mission line is practical while the rescaling of the pulse is purely mathematical. Because the stationary operation of the rescaled pulse is logically easy to perceive, we shall focus first on the rescaling of the pulse to study the fundamental properties of the SRP.

Pulse propagation in optical fiber is governed by the NLS equation,

$$i\frac{\partial u}{\partial z}+\frac{d(z)}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u=0, \tag{2}$$

where u, z, and t are the complex envelope of electric field, length, and time, respectively, and all of them are normalized. $d(z)$ represents fiber dispersion varying along the length z. We consider a two-sectional function $d(z)$ as a period of a transmission line as follows:

$$d(z)=\begin{cases} d_1 \sim O(\varepsilon) & (0 \leqslant z < z_1); \quad z_1 \sim O(1) \\ d_2 \sim O(1/\varepsilon) & (z_1 \leqslant z < z_2); \quad z_2 - z_1 \sim O(\varepsilon), \end{cases} \tag{3}$$

where $d_2 > 0$ — that is, the pulse receives anomalous dispersion effect in the second section of a period of the transmission line. We assume that an optical pulse has the amplitude and the width of $O(1)$, which means that in the first section of a period of the transmission line ($0 \leqslant z < z_1$), the orders of the accumulated SPM and dispersion effects of the pulse are $O(1)$ and $O(\varepsilon)$, respectively. On the other hand, in the second section ($z_1 \leqslant z < z_2$), the accumulated SPM and anomalous dispersion effects are in $O(\varepsilon)$ and $O(1)$, respectively. Thus the transmission line $d(z)$ we are discussing here is distinct from ones used in the frameworks of the GC theory or the DM theory in a sense of the magnitude of the accumulated nonlinearity and dispersion.

We adopted the so-called averaging method [12] for finding the SRP of the transmission line defined in Eq. (3). We used $u(t)=P_0^{1/2}$ sech t as the input pulse of the averaging procedure, where P_0 represents the peak power of the pulse. Specific considerations for the method in this case are the following two issues: (i) The pulse after propagating through every period of the line is rescaled by using Eq. (1) with a rescaling constant R. (ii) Instead of fixing the energy of the pulse for better convergence as done in Ref. [12], the peak power of the pulse is fixed to the initial value P_0 at each step of the procedure. This also compensates a numerical error, that is, the loss of the pulse energy due to the rescaling of the time in finite time window.

We show an example of the converged SRP when we choose $d_1=-0.0115$, $d_2=16.75$, $z_1=1.2$, and $z_2=1.215$ for $d(z)$, and $P_0=2, R=2.1$ for the peak power and the rescaling constant of the pulse, respectively. The power profile and the instantaneous frequency of the SRP are plotted as the lower and upper curves in Fig. 1(a), respectively. The instantaneous frequency is defined as $\delta\omega(t)=-\partial\theta(t)/\partial t$, where $\theta(t)$ is the phase of the pulse. $u_{\rm in}(t)$ and $u_{\rm out}(t)$ represent the SRP observed at $z=0$ and $z=z_2$ in the transmission line defined in Eq. (3), respectively, and $u_{\rm res}(t)$ is the rescaled pulse of $u_{\rm out}(t)$ using Eq. (1). Figure 1(b) shows the same power profiles in logarithmic scale. The pulse has a Gaussian-like main lobe

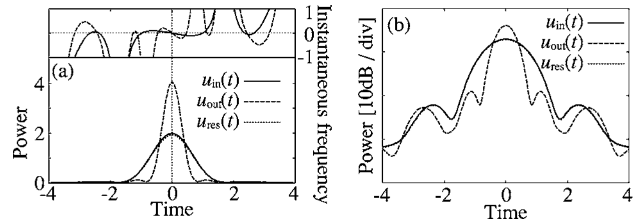

FIG. 1. (a) Power profiles (lower) and instantaneous frequencies (upper) of a SRP having $P_0=2$ and $R=2.1$. (b) The same power profiles shown in logarithmic scale. In both figures, $u_{\rm res}$ almost completely overlaps $u_{\rm in}$.

and oscillating tails. These characteristics are similar to the DM soliton. However, unlike the DM soliton, a nonlinear down chirp is observed in this case. It is also shown that $u_{\rm in}(t)$ and $u_{\rm res}(t)$ coincide with each other, where the slight mismatch of the amplitudes may originate from a numerical error associated with the rescaling of the pulse in the time.

We examined the stability of the SRP and numerically confirmed that the SRP could propagate several thousands periods of $d(z)$ in a periodic stationary manner. We also note that even if the input pulse has slightly different wave form from the exact stationary one, it converges on the stationary one during its propagation. This property is also confirmed by an experimental result as shown below.

There are two factors characterizing the SRP in a fixed transmission line, that are the peak power P_0 and the rescaling constant R. We have found that we are able to obtain stationary rescaled pulses having the same P_0 and different R.

Figure 2 shows the power profiles and the instantaneous frequencies of stationary rescaled pulses having the fixed peak power of $P_0=2$ and the rescaling constants $R=2.4$, 2.0 , 1.6, and 1.2. The full width at half maximum (FWHM) pulse widths Δt of the pulses are 1.77, 1.43, 1.22, and 1.04, respectively. The linear chirp parameters, which is defined as $C=(\partial/\partial t)\delta\omega|_{t=0}\Delta t^2/(4\ln 2)$, are $C=0.0910$, -0.246, -0.520, and -0.853 for each case. $d(z)$ is the same as that used for obtaining the result of Fig. 1.

To qualitatively grasp the propagation characteristics of the SRP, we investigate the motion of the pulse width Δt and the linear chirp parameter C within a period of $d(z)$. Figure 3

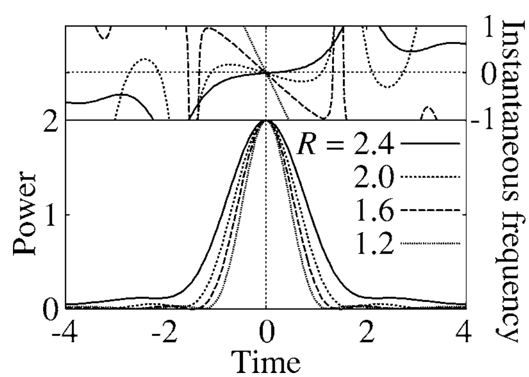

FIG. 2. Power profiles (lower) and instantaneous frequencies (upper) of SRPs having the fixed peak power $P_0=2$ for various rescaling constants R.

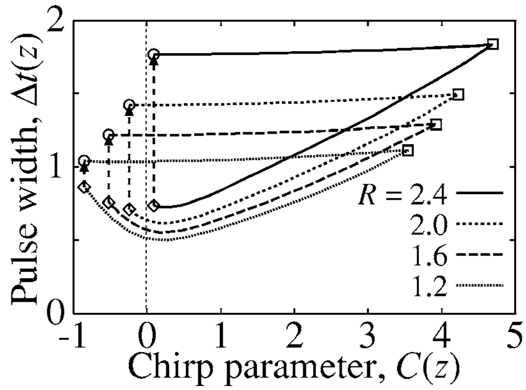

FIG. 3. Trajectories of $C(z)$ and $\Delta t(z)$ in a period of $d(z)$. The circles, squares, and diamonds correspond to $z=0$, z_1, and z_2, respectively. The dotted arrow means the rescaling of Eq. (1) for each pulse.

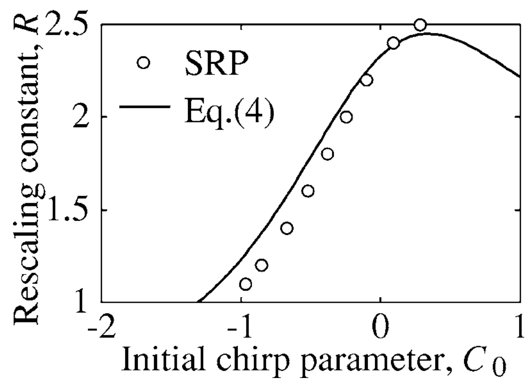

FIG. 4. The rescaling constants R vs the initial chirp parameters C_0 of SRPs with $P_0=2$ (points). The solid curve is calculated from Eq. (4) with $\varphi_{\rm NL}=2.4$.

depicts the trajectories of $\Delta t(z)$ and $C(z)$ of the SRPs shown in Fig. 2. The dotted arrow represents the rescaling of pulse at $z=z_2$. For all the cases in Fig. 3, $C_0=C_2$ and $\Delta t_0=R\Delta t_2$, where $C_n\equiv C(z_n)$, $\Delta t_n\equiv\Delta t(z_n)$, and $z_0=0$. The operation of the rescaling makes the trajectory "closed" and the pulse then recovers the initial wave form. In Fig. 3, $\Delta C\equiv C_1-C_0=C_1-C_2$ is almost the same for all the cases, because the amount of the accumulated SPM at $0\leqslant z<z_1$ is the same. A SRP with a small R should have a small C_1 because a large value of C_1 means a large spectral broadening of the pulse, i.e., a large rescaling constant R. Consequently, a SRP with a small R has a large negative value of C_0 under the condition of the same ΔC. On the other hand, at $z_1\leqslant z<z_2$, the up chirp of the pulse is compensated by the anomalous dispersion effect, $D\equiv d_2(z_2-z_1)/\Delta t_1^2\simeq d_2(z_2-z_1)/\Delta t_0^2$, and the chirp restores the initial value such that $C_2=C_0$. According to the linear pulse propagation theory, $\Delta C=(4\ln 2)D(1+C_1^2)\simeq(4\ln 2)d_2(z_2-z_1)[1+(C_0+\Delta C)^2]/\Delta t_0^2$ for a Gaussian pulse. This suggests that a SRP with a smaller R and a larger negative value of C_0 should have a smaller Δt_0 for the fixed ΔC. As another example, we have also obtained the SRPs with a fixed rescaling constant and varied peak powers. Similar to the case of the fixed P_0 and varied R, a SRP with a larger P_0 has a larger negative value of C_0 and a smaller Δt_0. On the other hand, considering an application of SRP to design of pulse compressors, it is effective to vary P_0 and R simultaneously. For example, a SRP with small P_0 and $R\sim 1$ has a sechlike wave form and a small chirp, in contrast to Fig. 1.

We then analytically estimate the operating parameter range of the SRP. We introduce a Gaussian ansatz with the linear chirp parameter as $u(0,t)=P_0^{1/2}\exp[-b_0^2t^2(1+iC_0)/2]$, where $b_0=2\sqrt{\ln 2}/\Delta t_0$. The spectral rms width of the pulse, $\Delta\omega$, can be analytically derived in the presence of the SPM effect unless dispersion effect is considered [13]. We then approximately obtain the rescaling constant R from the spectral broadening ratio of the pulse at $z=z_1$ as follows:

$$R\sim\frac{\Delta\omega(z_1)}{\Delta\omega(0)}=\left[1+\frac{\sqrt{2}C_0\varphi_{\rm NL}+(4/3\sqrt{3})\varphi_{\rm NL}^2}{1+C_0^2}\right]^{1/2}, \quad (4)$$

where $\varphi_{\rm NL}=P_0z_1$.

Figure 4 shows the relation between the rescaling constants R and the linear chirp parameters C_0 of the SRPs with $P_0=2$. The points are the numerical results taken from Fig. 2, plotted along with the analytically predicted curve from Eq. (4) with $\varphi_{\rm NL}=2.4$. In Fig. 4, the analytical prediction agrees well with the numerical results. Equation (4) indicates that R depends on $\varphi_{\rm NL}$ and C_0 but not on b_0. A significant insight obtained from Eq. (4) is that there exists the maximum of R and it is determined by $\varphi_{\rm NL}$ and C_0. This corresponds to the fact that the magnitude of SPM determines the spectral broadening ratio and therefore the compression ratio of the temporal width of the pulse. In fact, any SRP could not be obtained for the cases of $R>2.5$ under the condition of $P_0=2$ and $\varphi_{\rm NL}=2.4$. On the other hand, Eq. (4) indicates a possibility of the existence of the SRP with $R\leqslant 1$. Note that $R=1$ corresponds to the split-step soliton [3,10]. However, our numerical method for obtaining the SRP did not find any SRP with $R\leqslant 1$.

Using the rescaling of pulse, we have discussed the existence and the properties of the SRP so far. Indeed, the rescaling of Eq. (1) is equivalent to the rescaling of the transmission line. Replacing $u(t)$ and $d(z)$ in Eq. (2) by $R^{1/2}u(Rt)$ and $d'(z)$, and introducing new coordinates $\zeta=Rz$ and $\tau=Rt$, we obtain the following equation:

$$i\frac{\partial u}{\partial\zeta}+\frac{d'(\zeta/R)R}{2}\frac{\partial^2u}{\partial\tau^2}+|u|^2u=0, \quad (5)$$

for $u(\zeta,\tau)$. If $d'(\zeta/R)R=d(\zeta)$, Eqs. (2) and (5) are identical, and the pulse propagation characteristics are equivalent to each other in their own coordinates z,t, and ζ,τ. In other words, the behavior of pulse $R^{1/2}u(Rt)$ in the transmission line $d(Rz)/R$ is equivalent to that of $u(t)$ in $d(z)$. We then consider a concatenation of the rescaled transmission lines. The length and dispersion of fibers in the nth period of such lines shall be rescaled as $z_{2n-1}-z_{2n-2}=z_1/R^{n-1}$, $z_{2n}-z_{2n-1}=(z_2-z_1)/R^{n-1}$, $d_{2n-1}=d_1/R^{n-1}$, and $d_{2n}=d_2/R^{n-1}$ for $n\geqslant 2$, respectively. If we assume that the pulse linearly evolves in anomalous dispersion fibers, we can keep fiber dispersion as $d_{2n}=d_2$ for all periods. The lengths of anomalous dispersion fibers are then modified to $z_{2n}-z_{2n-1}=(z_2-z_1)/R^{2(n-1)}$. When the initial pulse $u_0(t)$ is a SRP of the first period of the line, the pulse will propagate over the line being repeatedly compressed by the ratio of R, and the wave form of the pulse at $z=z_{2n}$ is $u_n(t)=R^{n/2}u_0(R^n t)$ for $n \geqslant 1$. Note that such propagation could be regarded as "discrete self-similar" propagation, in contrast with "continuous self-similar" one [14,15].

We have conducted an experiment in order to observe a SRP in a periodically rescaled transmission line shown above. We consider the SRP shown in Fig. 1, that is, $P_0=2$ and $R=2.1$ for the pulse parameters, and $d_1=-0.0115$, $d_2=16.75$, $z_1=1.2$, and $z_2=1.215$ for the transmission line $d(z)$ in the normalized space. The number of periods of periodically rescaled transmission line is four, and each period in the physical space consists of concatenations of a highly nonlinear fiber (dispersion -0.3 ps/nm/km and nonlinearity 19.8 W^{-1} km^{-1}) and a single-mode fiber (16.3 ps/nm/km and 1.3 W^{-1} km^{-1}). As an input, we use a 40 GHz repeating out-of-phase sechlike pulse sequence with the 7 ps FWHM and the 0.357 W peak power, which is generated from the 40 GHz-spaced two-mode compressed through a comblike profiled fiber [16]. To realize the conditions for the SRP and the transmission line, we set in the physical space the lengths of the fibers in the first period of the line as $L_{\mathrm{HNLF}}^{(1)}=340$ m and $L_{\mathrm{SMF}}^{(1)}=285.5$ m. The lengths of the fibers in the succeeding nth period are then given by $L_{\mathrm{HNLF}}^{(n)}=L_{\mathrm{HNLF}}^{(1)}/R^{n-1}$ and $L_{\mathrm{SMF}}^{(n)}=L_{\mathrm{SMF}}^{(1)}/R^{2(n-1)}$. The expected FWHM of the SRP after propagating nth period of the line is $\Delta T_n=\Delta T_0/R^n=7\times 2.1^{-n}$ ps, that is $\Delta T_4=0.36$ ps. The experimental result was $\Delta T_1=3.24$, $\Delta T_2=1.49$, $\Delta T_3=0.767$, and $\Delta T_4=0.375$ ps, which agrees well with the prediction. Figure 5 shows (a) the autocorrelation traces and (b) the spectra of the experimentally observed pulse at the end of the fourth period of the line, as well as the numerically predicted SRP. In both figures, the experimental result agrees well with the SRP. These results confirm that an optical sechlike input pulse having the parameters and the wave form close to a SRP of a periodically rescaled transmission line approaches to the SRP during its propagation.

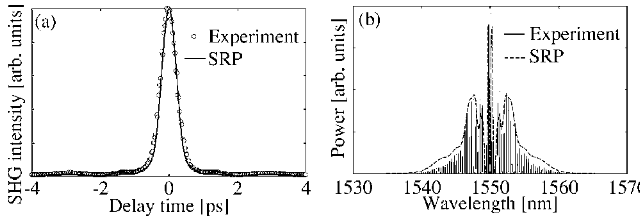

FIG. 5. (a) Autocorrelation traces and (b) optical spectra of the experimentally observed pulse sequence and the SRP.

In conclusion, we have discovered a nonlinear stationary rescaled pulse in a transmission line where the SPM and the anomalous dispersion alternately accumulate up to an extensive magnitude of $O(1)$. This pulse is distinct from the GC soliton, the DM soliton, or the split-step soliton in view of the accumulated value of the perturbations. We have investigated the characteristics of the SRP and clarified the operating parameter range. Applying the rescaling to a transmission line rather than to a pulse, we have proposed a method for realizing the SRP propagation in fibers, which is applicable to optical pulse compressors. We have also experimentally verified a sechlike pulse approaches to the SRP in a periodically rescaled transmission line.

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[1] A. Hasegawa and Y. Kodama, Opt. Lett. **15**, 1443 (1990).
[2] N. J. Smith, F. M. Knox, N. J. Doran, K. J. Blow, and I. Bennion, Electron. Lett. **32**, 54 (1996).
[3] R. Driben and B. A. Malomed, Opt. Commun. **185**, 439 (2000).
[4] J. M. Dudley, A. C. Peacock, and G. Millot, Opt. Commun. **193**, 253 (2001).
[5] H. A. Haus, IEEE J. Sel. Top. Quantum Electron. **6**, 1173 (2000).
[6] M. J. Ablowitz, B. Ilan, and S. T. Cundiff, Opt. Lett. **29**, 1808 (2004).
[7] A. Hasegawa and Y. Kodama, Phys. Rev. Lett. **66**, 161 (1991).
[8] M. J. Ablowitz, T. Hirooka, and T. Inoue, J. Opt. Soc. Am. B **19**, 2876 (2002), and Refs. [5–7] therein.
[9] S. V. Chernikov, J. R. Taylor, and R. Kashyap, Opt. Lett. **19**, 539 (1994).
[10] R. Driben, B. A. Malomed, and P. L. Chu, J. Opt. Soc. Am. B **20**, 2338 (2003).
[11] R. Driben, B. A. Malomed, and P. L. Chu, Opt. Commun. **245**, 227 (2005).
[12] J. H. B. Nijhof, W. Forysiak, and N. J. Doran, IEEE J. Sel. Top. Quantum Electron. **6**, 330 (2000); V. Cautaerts, A. Maruta, and Y. Kodama, Chaos **10**, 515 (2000); P. Y. P. Chen, P. L. Chu, and B. A. Malomed, Opt. Commun. **245**, 425 (2005).
[13] S. C. Pinault and M. J. Potasek, J. Opt. Soc. Am. B **2**, 1318 (1985).
[14] M. E. Fermann, V. I. Kruglov, B. C. Thomsen, J. M. Dudley, and J. D. Harvey, Phys. Rev. Lett. **84**, 6010 (2000).
[15] V. I. Kruglov, A. C. Peacock, and J. D. Harvey, Phys. Rev. Lett. **90**, 113902 (2003).
[16] K. Igarashi, S. Takasaka, R. Miyabe, J. Hiroishi, R. Sugizaki, T. Yagi, and S. Namiki, Tech. Dig. CLEO2004, paper CFC2, 2004 (unpublished).